

Comparison of Two Interpretations of Josephson Effect

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This paper puts forward an interpretation of the Josephson effect based on the Alternative Theory of Superconductivity (ATS). A comparison of ATS- and BCS-based interpretations is provided. It is demonstrated that the ATS-based interpretation, unlike that based on BCS theory, does not require a revision of fundamentals of quantum physics.

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I. INTRODUCTION

The Josephson effect holds a special place in theoretical physics. It could hardly be denied that it was the prediction [1] of this effect, as well as its subsequent interpretation [2] and observation [3, 4], that made a stunning impression on the contemporary academic community, leaving it with no further doubts as to the validity of the BCS theory.

Reasons to question the correctness of that theory have not arisen till much later, when high-temperature superconductors were discovered. Up to that moment, it was believed based on BCS theory's estimates, that maximum values of transition temperatures should lie in the vicinity of 40 K. Although some attempts to suggest alternative versions of explanation of superconductivity phenomena have been made late in the last century [5, 6, 7, 8, 9], these attempts were received by most theorists with skepticism. That is why currently, an overwhelming majority of works that are taken seriously are related to BCS theory, and the existing discussions are mostly carried out at the level of determination of appearance mechanisms of strong electron-electron attraction.

It would be interesting to trace the connections between the BCS theory and other concepts by taking the example of the BCS-BEC crossover [10], which, according to the authors, should take place when Cooper pair binding energy values are comparable with Fermi energies of electrons. It should be noted that the idea of describing the superconductivity phenomenon using the possibility of Bose-Einstein condensation (BEC) of electron pairs appeared as such before Cooper [11, 12], but the linear relationship between the zero-temperature energy gap Δ and the transition temperature T_c of then-known superconductors was difficult to interpret in the framework of that concept. Therefore, the present-day revival of this idea is naturally related to the BCS theory.

Curiously enough, the said linear relationship can be explained in the framework of the Alternative Theory of Superconductivity (ATS), unrelated to the BCS theory [13]. One should then note that calculations of transition temperatures in the mean value approximation are only of illustrative nature in physics, and not expected to provide high-precision predictions [14]. From this point of view, if the two theories were to be compared, the relation $T_c \approx 2\Delta/3.5$ obtained from the BCS theory does not provide any advantages as compared with the ATS relation $T_c \approx \Delta/2$ [13].

As for the calculation of transition temperature values carried out in the framework of the ATS [15], it has unexpectedly revealed a clear advantage over the BCS theory. Indeed, both theories only use one adjustable parameter. However, in the BCS theory, this parameter is the so-called Coulomb pseudopotential [16], which is in no way related to any measurable parameter of the system, i.e. is practically "bare". On the other hand, in the ATS, the adjustable parameter should be on the order of average phonon frequency, and numerical calculation results do confirm this theoretical prediction [15]. It is quite obvious that, had this study appeared in 1960s, physics of superconductivity could have an entirely different history. However, what with the enormous bulk of experimental data interpreted since the first paper on BCS theory appeared [17], rejecting the results of a gigantic work done by several generations of theorists constitutes a huge psychological problem. In this situation, ATS partisans find themselves constrained to select one experiment after another to compare their interpretations in both theories in the hope of finding differences of interpretation that would be significant from the point of view of selection of the right theory. In this paper, the Josephson effect was selected as such an experiment bearing in mind the key role it had played for recognition of the BCS theory as the standard theory of superconductivity.

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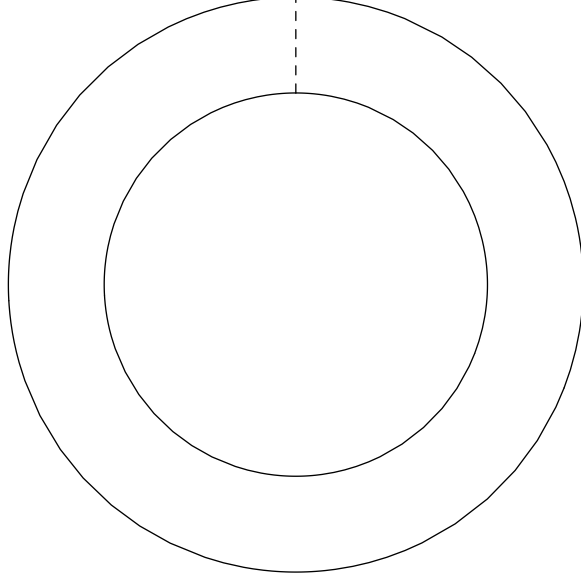


FIG. 1: Geometric setup of the system under consideration. The dashed line represents the ring gap.

II. CONTACT BETWEEN TWO IDENTICAL NORMAL METALS

We start examining the problem by formulating the Hamiltonian of a tunnel junction in the presence of an electromagnetic field of the type that can only be taken into account using a gradient transformation. An example of such system can be suggested in the form of a metal ring with a gap as shown in Fig. 1. When magnetic flux Φ passes through the ring, its effect can be totally taken into account using a gradient transformation, the phase difference φ of the gradient transformation on both sides of the gap being linearly dependent on Φ , i.e. $\varphi = 2\pi\Phi/\Phi_0$, where $\Phi_0 = hc/e$. In all subsequent calculations, we take $\hbar = 1$, as usual.

Consider a cubic lattice of dimensions $L_x \times L_y \times L_z$. Let $c_{\sigma\mathbf{g}}^+$ and $c_{\sigma\mathbf{g}}$ be, respectively, creation and annihilation operators of an electron with a spin index $\sigma = \uparrow, \downarrow$ in the orbital state, which forms the system conduction band, at a node \mathbf{g} ; then, for \mathbf{g} :

$$\begin{aligned} \mathbf{g}_\alpha &= 1..N_\alpha, \\ N_\alpha &= L_\alpha/a + 1, \end{aligned} \quad (1)$$

where $\alpha = x, y, z$, and a is the elementary cell size. Such a system can provide a good description of the ring when $L_x \gg L_y, L_z$. In this case L_y and L_z define the ring thickness, and L_x is equal to the ring perimeter.

Transformation into momentum representation defined with creation operator $c_{\sigma\mathbf{p}}^+$ and annihilation operator $c_{\sigma\mathbf{p}}$ of an electron carrying a momentum \mathbf{p} is performed in a standard way:

$$\begin{aligned} c_{\sigma\mathbf{p}}^+ &= N_0^{-1/2} \sum_{\mathbf{g}} \exp(i\varphi(\mathbf{a}\mathbf{g})) \sin(\mathbf{p}_x \mathbf{a} \mathbf{g}_x) \sin(\mathbf{p}_y \mathbf{a} \mathbf{g}_y) \sin(\mathbf{p}_z \mathbf{a} \mathbf{g}_z) c_{\sigma\mathbf{g}}^+, \\ c_{\sigma\mathbf{p}} &= N_0^{-1/2} \sum_{\mathbf{g}} \exp(-i\varphi(\mathbf{a}\mathbf{g})) \sin(\mathbf{p}_x \mathbf{a} \mathbf{g}_x) \sin(\mathbf{p}_y \mathbf{a} \mathbf{g}_y) \sin(\mathbf{p}_z \mathbf{a} \mathbf{g}_z) c_{\sigma\mathbf{g}}, \end{aligned} \quad (2)$$

where $\mathbf{p}_\alpha = \frac{\pi \bar{\mathbf{p}}_\alpha}{L_\alpha + 2a}$, $\bar{\mathbf{p}}_\alpha = 1..N_\alpha$, $N_0 = \prod_{\alpha} N_{0\alpha}$, $N_{0\alpha} = \frac{L_\alpha + 2a}{2a}$, and $\varphi(\mathbf{a}\mathbf{g})$ is the phase of the gradient transformation.

In this form, operators $c_{\sigma\mathbf{p}}^+$ and $c_{\sigma\mathbf{p}}$ correspond to eigenstates of the system. The reverse transformation into the node representation is defined as follows:

$$\begin{aligned} c_{\sigma\mathbf{g}}^+ &= N_0^{-1/2} \exp(-i\varphi(\mathbf{a}\mathbf{g})) \sum_{\mathbf{p}} \sin(\mathbf{p}_x \mathbf{a} \mathbf{g}_x) \sin(\mathbf{p}_y \mathbf{a} \mathbf{g}_y) \sin(\mathbf{p}_z \mathbf{a} \mathbf{g}_z) c_{\sigma\mathbf{p}}^+, \\ c_{\sigma\mathbf{g}} &= N_0^{-1/2} \exp(i\varphi(\mathbf{a}\mathbf{g})) \sum_{\mathbf{p}} \sin(\mathbf{p}_x \mathbf{a} \mathbf{g}_x) \sin(\mathbf{p}_y \mathbf{a} \mathbf{g}_y) \sin(\mathbf{p}_z \mathbf{a} \mathbf{g}_z) c_{\sigma\mathbf{p}}. \end{aligned} \quad (3)$$

Up to now, we considered the gap to be an ideal electric isolator. Now, take into account the possibility of electron transfer between the adjacent nodes on different sides of the gaps; to do so, we introduce the tunneling Hamiltonian

\hat{H}_t :

$$\hat{H}_t = -\gamma \sum_{\sigma} \sum_{\mathbf{g}_y, \mathbf{g}_z} c_{\sigma\{1, \mathbf{g}_y, \mathbf{g}_z\}}^+ c_{\sigma\{N_x, \mathbf{g}_y, \mathbf{g}_z\}} + H.c. \quad (4)$$

Here, apparently, γ is the hybridization potential, which we take to be a real value measured in energy units. One can easily obtain an expression for \hat{H}_t in momentum representation:

$$\hat{H}_t = \gamma e^{i\varphi} N_{0x}^{-1} \sum_{\sigma} \sum_{\mathbf{p}, \mathbf{k}} (-1)^{\bar{\mathbf{k}}_x} \delta_{\mathbf{k}_y}^{\bar{\mathbf{p}}_y} \delta_{\mathbf{k}_z}^{\bar{\mathbf{p}}_z} \sin(\mathbf{p}_x a) \sin(\mathbf{k}_x a) c_{\sigma\mathbf{p}}^+ c_{\sigma\mathbf{k}} + H.c. \quad (5)$$

where φ is the phase difference between the sides of the contact, i.e. $\varphi = \varphi(aN_x, a\mathbf{g}_y, a\mathbf{g}_z) - \varphi(a, a\mathbf{g}_y, a\mathbf{g}_z)$, and $\delta_{\mathbf{k}_\alpha}^{\bar{\mathbf{p}}_\alpha}$ is the Kronecker delta. This expression can be transformed so as to make the presence of the Josephson effect evident even in a normal metal, although with a certain reserve:

$$\hat{H}_t = N_{0x}^{-1} \sum_{\sigma} \sum_{\mathbf{p}, \mathbf{k}} \left[(-1)^{\bar{\mathbf{k}}_x} e^{i\varphi} + (-1)^{\bar{\mathbf{p}}_x} e^{-i\varphi} \right] \lambda(\mathbf{p}, \mathbf{k}) c_{\sigma\mathbf{p}}^+ c_{\sigma\mathbf{k}}, \quad (6)$$

where $\lambda(\mathbf{p}, \mathbf{k}) = \gamma \delta_{\mathbf{k}_y}^{\bar{\mathbf{p}}_y} \delta_{\mathbf{k}_z}^{\bar{\mathbf{p}}_z} \sin(\mathbf{p}_x a) \sin(\mathbf{k}_x a)$.

Change in the system energy $\delta E^{(1)}(\varphi)$ is in the first order of perturbation theory over λ defined by the expression:

$$\delta E^{(1)}(\varphi) = 2N_{0x}^{-1} \cos(\varphi) \sum_{\sigma, \mathbf{p}} (-1)^{\bar{\mathbf{p}}_x} \lambda(\mathbf{p}, \mathbf{p}) n_{\sigma\mathbf{p}}. \quad (7)$$

A simple analysis of this expression makes one conclude that, at certain relations between L_x , L_y and L_z the minimum of the energy system can be displaced to a position of $\varphi = \pi$, but the energy gain obtained in transition to this state tends to zero as the system size increases, i.e. one deals with a so-called quantum dimension effect [18]. This effect can be used in attempts of creating matrices featuring unusual magnetic properties.

As for the second order of perturbation theory, it yields the following expression for the correction $\delta E^{(2)}(\varphi)$:

$$\delta E^{(2)}(\varphi) = \frac{1}{2} N_{0x}^{-2} \sum_{\sigma} \sum_{\mathbf{p} \neq \mathbf{k}} \left| (-1)^{\bar{\mathbf{k}}_x} e^{i\varphi} + (-1)^{\bar{\mathbf{p}}_x} e^{-i\varphi} \right|^2 \lambda^2(\mathbf{p}, \mathbf{k}) \frac{n_{\sigma\mathbf{p}} - n_{\sigma\mathbf{k}}}{\varepsilon_{\mathbf{p}} - \varepsilon_{\mathbf{k}}}. \quad (8)$$

These corrections are related to the electron transition through the junction, which is why there exists the following obvious relation $\delta E(\varphi) - \delta E(0) = \int_0^t I(t') V(t') dt' = e^{-1} \int_0^\varphi I(\varphi') d\varphi'$, where e is the electron charge and V is the circuit e.m.f. This can easily be seen considering the tunneling current operator, which can be expressed as:

$$\hat{I}_t = -ie\gamma \sum_{\sigma} \sum_{\mathbf{g}_y, \mathbf{g}_z} c_{\sigma\{1, \mathbf{g}_y, \mathbf{g}_z\}}^+ c_{\sigma\{N_x, \mathbf{g}_y, \mathbf{g}_z\}} + H.c. \quad (9)$$

or, in the momentum representation, as:

$$\hat{I}_t = -ieN_{0x}^{-1} \sum_{\sigma} \sum_{\mathbf{p}, \mathbf{k}} \left[(-1)^{\bar{\mathbf{k}}_x} e^{-i\varphi} - (-1)^{\bar{\mathbf{p}}_x} e^{i\varphi} \right] \lambda(\mathbf{p}, \mathbf{k}) c_{\sigma\mathbf{k}}^+ c_{\sigma\mathbf{p}}. \quad (10)$$

Indeed, corrections of the first and second order over λ to a current averaged over the ground state are of the form of:

$$\begin{aligned} I^{(1)}(\varphi) &= -2eN_{0x}^{-1} \sin(\varphi) \sum_{\sigma, \mathbf{p}} (-1)^{\bar{\mathbf{p}}_x} \lambda(\mathbf{p}, \mathbf{p}) n_{\sigma\mathbf{p}}, \\ I^{(2)}(\varphi) &= -2e \sin(2\varphi) N_{0x}^{-2} \sum_{\sigma} \sum_{\mathbf{p} \neq \mathbf{k}} (-1)^{\bar{\mathbf{p}}_x - \bar{\mathbf{k}}_x} \lambda^2(\mathbf{p}, \mathbf{k}) \frac{n_{\sigma\mathbf{p}} - n_{\sigma\mathbf{k}}}{\varepsilon_{\mathbf{p}} - \varepsilon_{\mathbf{k}}}, \end{aligned} \quad (11)$$

where $\varepsilon_{\mathbf{p}}$ and $\varepsilon_{\mathbf{k}}$ are energies of electrons carrying, respectively, momenta \mathbf{p} and \mathbf{k} . The obtained expressions confirm the validity of the above relation, which links the system energy change to the current running through the contact.

It becomes evident that the Josephson effect is present even in a normal metal, although it is so small in macroscopic samples that one actually deals with an oscillating series with terms of similar magnitude.

Curiously enough, when only the second-order correction $\delta E^{(2)}(\varphi)$ is taken into account, Josephson transition for a system of two normal metals is at zero temperature a so-called π -type junction since $\delta E^{(2)}(\varphi)$ is at a minimum at $\varphi = \pi/2$ (in the BCS theory, the parameter φ has historically been defined to be equal to $2e \int_0^t V(t') dt'$). Indeed, when the values of $\bar{\mathbf{p}}_y$ and $\bar{\mathbf{p}}_z$ are fixed, transition from filled to unfilled states takes place with a change in parity of the value of $\bar{\mathbf{p}}_x$. That means that the energy-related denominator in the above expression is at a minimum when $(-1)^{\bar{\mathbf{p}}_x - \bar{\mathbf{k}}_x} = -1$, which brings about this conclusion.

III. CONTACT BETWEEN TWO IDENTICAL SUPERCONDUCTORS IN THE ATS

Now consider a ring made of a superconductor. In the ATS, a superconductor differs from a normal metal in that its effective electron-electron interaction is strong enough to enable production of coupled-pair states which, however, have nothing in common with Cooper pairs. Presence of coupled states brings about appearance of a gap in the single-particle excitation spectrum. Therefore, considering the dynamics of a superconductor in single-particle approximation is meaningless. Indeed, the spectrum gap produces a delta dependence of group velocity of electrons on momentum, which makes it difficult to describe the interaction of the system with an electromagnetic field.

To describe the system in two-particle approximation, it is convenient to use a formalism related to a transition to new electron creation and annihilation operators [15], respectively, C^+ and C . From the mathematical point of view, these operators have much in common with those introduced by Kadanoff and Martin in their dynamical generalization of the static BCS approach [19]. A significant difference between the approaches is only revealed at the physical level, when correlators introduced by Kadanoff and Martin are defined.

The new representation allows easily obtaining the ground-state wave function, as well as spectra of single- and two-particle excitations. Parent operators c^+ and c are related to the operators C^+ and C by the following expressions:

$$\begin{aligned} c_{\sigma\mathbf{p}}^+ &= C_{\sigma\mathbf{p}}^+ + \sum_{\nu} \sum_{\mathbf{k}, \mathbf{q}} \theta_{\sigma, \nu, \mathbf{q}}^{*\mathbf{k}-\mathbf{q}, \mathbf{p}+\mathbf{q}} C_{\sigma\mathbf{p}+\mathbf{q}}^+ C_{\nu\mathbf{k}-\mathbf{q}}^+ C_{\nu\mathbf{k}}, \\ c_{\sigma\mathbf{p}} &= C_{\sigma\mathbf{p}} + \sum_{\nu} \sum_{\mathbf{k}, \mathbf{q}} \theta_{\sigma, \nu, \mathbf{q}}^{\mathbf{k}-\mathbf{q}, \mathbf{p}+\mathbf{q}} C_{\nu\mathbf{k}}^+ C_{\nu\mathbf{k}-\mathbf{q}} C_{\sigma\mathbf{p}+\mathbf{q}}, \end{aligned} \quad (12)$$

where

$$\begin{aligned} \theta_{\sigma, \sigma, \mathbf{q}}^{\mathbf{p}, \mathbf{k}} &= \frac{1}{2} \left(\delta_{\mathbf{k}-\bar{\mathbf{p}}}^{\bar{\mathbf{q}}} - \delta_{\bar{\mathbf{q}}}^0 \right) + \chi_{1, \mathbf{q}}^{\mathbf{p}, \mathbf{k}}, \\ \theta_{\sigma, -\sigma, \mathbf{q}}^{\mathbf{p}, \mathbf{k}} &= -\delta_{\bar{\mathbf{q}}}^0 + \chi_{0, \mathbf{q}}^{\mathbf{p}, \mathbf{k}} + \chi_{1, \mathbf{q}}^{\mathbf{p}, \mathbf{k}}, \end{aligned} \quad (13)$$

$\delta_{\mathbf{n}}^{\mathbf{m}}$ is the 3D Kronecker delta, i.e. $\delta_{\mathbf{n}}^{\mathbf{m}} = \prod_{\alpha} \delta_{\mathbf{n}_{\alpha}}^{\mathbf{m}_{\alpha}}$, and $\chi_{0, \mathbf{q}}^{\mathbf{p}, \mathbf{k}}$ and $\chi_{1, \mathbf{q}}^{\mathbf{p}, \mathbf{k}}$ are solutions of eigenvalue problems for two-particle states with spins 0 and 1, respectively.

The following commutation relations are valid for $\chi_{0, \mathbf{q}}^{\mathbf{p}, \mathbf{k}}$ and $\chi_{1, \mathbf{q}}^{\mathbf{p}, \mathbf{k}}$ [20]:

$$\begin{aligned} \chi_{0, \mathbf{q}}^{\mathbf{p}, \mathbf{k}} &= \chi_{0, \mathbf{k}-\mathbf{p}-\mathbf{q}}^{\mathbf{p}, \mathbf{k}}, \chi_{0, \mathbf{q}}^{\mathbf{p}, \mathbf{k}} = \chi_{0, -\mathbf{q}}^{\mathbf{k}, \mathbf{p}}, \\ \chi_{1, \mathbf{q}}^{\mathbf{p}, \mathbf{k}} &= -\chi_{1, \mathbf{k}-\mathbf{p}-\mathbf{q}}^{\mathbf{p}, \mathbf{k}}, \chi_{1, \mathbf{q}}^{\mathbf{p}, \mathbf{k}} = \chi_{1, -\mathbf{q}}^{\mathbf{k}, \mathbf{p}}. \end{aligned} \quad (14)$$

As for normalizing relations, they have the following form:

$$\begin{aligned} \sum_{\mathbf{x}} \chi_{0, \mathbf{x}}^{\mathbf{p}, \mathbf{k}+\mathbf{q}} \chi_{0, \mathbf{q}-\mathbf{x}}^{\mathbf{k}, \mathbf{p}+\mathbf{q}} &= \frac{1}{2} \left(\delta_{\bar{\mathbf{q}}}^0 + \delta_{\bar{\mathbf{k}}}^{\bar{\mathbf{p}}} \right), \\ \sum_{\mathbf{x}} \chi_{1, \mathbf{x}}^{\mathbf{p}, \mathbf{k}+\mathbf{q}} \chi_{1, \mathbf{q}-\mathbf{x}}^{\mathbf{k}, \mathbf{p}+\mathbf{q}} &= \frac{1}{2} \left(\delta_{\bar{\mathbf{q}}}^0 - \delta_{\bar{\mathbf{k}}}^{\bar{\mathbf{p}}} \right). \end{aligned} \quad (15)$$

The Hamiltonian \hat{H}_t can now be expressed via the new operators C^+ and C^+ :

$$\begin{aligned} \hat{H}_t &= N_{0x}^{-1} \sum_{\mathbf{p}, \mathbf{p}'} \left[(-1)^{\bar{\mathbf{p}}'_x} e^{i\varphi} + (-1)^{\bar{\mathbf{p}}_x} e^{-i\varphi} \right] \lambda(\mathbf{p}, \mathbf{p}') \sum_{\sigma} C_{\sigma\mathbf{p}}^+ C_{\sigma\mathbf{p}'} \\ &\quad - N_{0x}^{-1} \sum_{\mathbf{p}, \mathbf{p}'} \left[(-1)^{\bar{\mathbf{p}}'_x} e^{i\varphi} + (-1)^{\bar{\mathbf{p}}_x} e^{-i\varphi} \right] \lambda(\mathbf{p}, \mathbf{p}') \sum_{\sigma, \nu, \mathbf{k}} C_{\sigma\mathbf{p}}^+ C_{\nu\mathbf{k}}^+ C_{\nu\mathbf{k}} C_{\sigma\mathbf{p}'} \\ &\quad + N_{0x}^{-1} \sum_{\mathbf{p}, \mathbf{p}'} \left[(-1)^{\bar{\mathbf{p}}'_x} e^{i\varphi} + (-1)^{\bar{\mathbf{p}}_x} e^{-i\varphi} \right] \lambda(\mathbf{p}, \mathbf{p}') \sum_{\sigma, \nu, \mathbf{k}, \mathbf{q}, \mathbf{q}'} \chi_{1, \mathbf{q}}^{\mathbf{k}-\mathbf{q}, \mathbf{p}+\mathbf{q}^*} \chi_{1, \mathbf{q}'}^{\mathbf{k}-\mathbf{q}', \mathbf{p}'+\mathbf{q}'} C_{\sigma\mathbf{p}+\mathbf{q}}^+ C_{\nu\mathbf{k}-\mathbf{q}}^+ C_{\nu\mathbf{k}-\mathbf{q}'} C_{\sigma\mathbf{p}'+\mathbf{q}'} \\ &\quad + N_{0x}^{-1} \sum_{\mathbf{p}, \mathbf{p}'} \left[(-1)^{\bar{\mathbf{p}}'_x} e^{i\varphi} + (-1)^{\bar{\mathbf{p}}_x} e^{-i\varphi} \right] \lambda(\mathbf{p}, \mathbf{p}') \sum_{\sigma, \mathbf{k}, \mathbf{q}, \mathbf{q}'} \chi_{0, \mathbf{q}}^{\mathbf{k}-\mathbf{q}, \mathbf{p}+\mathbf{q}^*} \chi_{0, \mathbf{q}'}^{\mathbf{k}-\mathbf{q}', \mathbf{p}'+\mathbf{q}'} C_{\sigma\mathbf{p}+\mathbf{q}}^+ C_{-\sigma\mathbf{k}-\mathbf{q}}^+ C_{-\sigma\mathbf{k}-\mathbf{q}'} C_{\sigma\mathbf{p}'+\mathbf{q}'} \end{aligned} \quad (16)$$

This expression does not include terms having the structure of $C^+ C^+ C^+ C C C$, since those should only be taken into account in three-particle approximation, which involves introducing additional terms into right hand side of (12).

One obtains for uncoupled states:

$$\begin{aligned} \chi_{0, \mathbf{q}}^{\mathbf{p}, \mathbf{k}} &\approx \frac{1}{2} \left(\delta_{\bar{\mathbf{q}}}^0 + \delta_{\bar{\mathbf{k}-\bar{\mathbf{p}}}}^{\bar{\mathbf{q}}} \right), \\ \chi_{1, \mathbf{q}}^{\mathbf{p}, \mathbf{k}} &\approx \frac{1}{2} \left(\delta_{\bar{\mathbf{q}}}^0 - \delta_{\bar{\mathbf{k}-\bar{\mathbf{p}}}}^{\bar{\mathbf{q}}} \right). \end{aligned} \quad (17)$$

The non-strict equalities in these expressions are related to corrections that are similar to Born approximation corrections in the scattering theory, which we neglect.

There is every reason to assume [15] that in the simplest low-temperature superconductors, coupled states are only produced for zero-spin states, either at a single node of the momentum lattice or at adjacent nodes. Then, one may hereafter consider the following relations to be satisfied:

$$\begin{aligned}\chi_{0,\mathbf{q}}^{\mathbf{p},\mathbf{k}} &= \frac{1}{2} \left(\delta_{\mathbf{q}}^0 + \delta_{\mathbf{k}-\mathbf{p}}^{\bar{\mathbf{q}}} \right) + h_{\mathbf{q}}^{\mathbf{p},\mathbf{k}}, \\ \chi_{1,\mathbf{q}}^{\mathbf{p},\mathbf{k}} &= \frac{1}{2} \left(\delta_{\mathbf{q}}^0 - \delta_{\mathbf{k}-\mathbf{p}}^{\bar{\mathbf{q}}} \right),\end{aligned}\quad (18)$$

$h_{\mathbf{q}}^{\mathbf{p},\mathbf{k}}$ is only non-zero when $|\mathbf{p}_\alpha - \mathbf{k}_\alpha| < 2$, and it only is in these cases that:

$$h_{\mathbf{q}}^{\mathbf{p},\mathbf{k}} = -\frac{1}{2} \left(\delta_{\mathbf{q}}^0 + \delta_{\mathbf{k}-\mathbf{p}}^{\bar{\mathbf{q}}} \right) + \Omega^{-1/2} \psi^{\mathbf{p},\mathbf{k}}(\mathbf{q}), \quad (19)$$

where $\psi^{\mathbf{p},\mathbf{k}}(\mathbf{q})$ is the coupled-state wave function, and $\Omega = \pi^{-3} \prod_{\alpha} L_{\alpha}$. If one believes the effective potential of the electron-electron interaction to be weakly dependent on electron momenta and spherically symmetric, the above normalization relations then yield:

$$\psi^{\mathbf{p},\mathbf{p}}(\mathbf{q}) \approx \psi_0(q), \quad (20)$$

and with $\mathbf{p} \neq \mathbf{k}$,

$$\psi^{\mathbf{p},\mathbf{k}}(\mathbf{q}) \approx \psi_0(q) / \sqrt{2}, \quad (21)$$

while $\int |\psi_0(q)|^2 d\mathbf{q} = 1$. Difference between the expressions (20) and (21) reflects the fact that two electrons of opposite spins located at the same node form a zero-spin state, and the probability of such a state to be formed at different nodes is equal to 1/2.

Expanding of the Hamiltonian \hat{H}_t in powers of $h_{\mathbf{q}}^{\mathbf{p},\mathbf{k}}$ reveals that it consists of four terms. The first term $\hat{H}_t^{(1)}$ simply reproduces the Hamiltonian of a normal metal, symbols C substituting c . Of course, the terms $\hat{H}_t^{(1)}$ yield an oscillating series with terms of similar magnitude and do not contribute to Josephson current for macroscopic systems. The second term $\hat{H}_t^{(2)}$ is related to Kronecker deltas contained in $h_{\mathbf{q}}^{\mathbf{p},\mathbf{k}}$ and has the form of:

$$\begin{aligned}\hat{H}_t^{(2)} &= -\frac{1}{4} N_{0x}^{-1} \sum_{|\mathbf{p}_\alpha - \mathbf{k}_\alpha| < 2} \sum_{\mathbf{p}'} \left[(-1)^{\bar{\mathbf{p}}'_x} e^{i\varphi} + (-1)^{\bar{\mathbf{p}}_x} e^{-i\varphi} \right] \lambda(\mathbf{p}, \mathbf{p}') \sum_{\sigma} C_{\sigma\mathbf{p}}^+ C_{-\sigma\mathbf{k}}^+ C_{-\sigma\mathbf{k}} C_{\sigma\mathbf{p}'} \\ &\quad -\frac{1}{4} N_{0x}^{-1} \sum_{|\mathbf{p}_\alpha - \mathbf{k}_\alpha| < 2} \sum_{\mathbf{p}'} \left[(-1)^{\bar{\mathbf{p}}'_x} e^{i\varphi} + (-1)^{\bar{\mathbf{p}}_x} e^{-i\varphi} \right] \lambda(\mathbf{p}, \mathbf{p}') \sum_{\sigma} C_{\sigma\mathbf{k}}^+ C_{-\sigma\mathbf{p}}^+ C_{-\sigma\mathbf{k}} C_{\sigma\mathbf{p}'} \\ &\quad -\frac{1}{4} N_{0x}^{-1} \sum_{\mathbf{p}} \sum_{|\mathbf{p}'_\alpha - \mathbf{k}'_\alpha| < 2} \left[(-1)^{\bar{\mathbf{p}}'_x} e^{i\varphi} + (-1)^{\bar{\mathbf{p}}_x} e^{-i\varphi} \right] \lambda(\mathbf{p}, \mathbf{p}') \sum_{\sigma} C_{\sigma\mathbf{p}}^+ C_{-\sigma\mathbf{k}'}^+ C_{-\sigma\mathbf{k}'} C_{\sigma\mathbf{p}'} \\ &\quad -\frac{1}{4} N_{0x}^{-1} \sum_{\mathbf{p}} \sum_{|\mathbf{p}'_\alpha - \mathbf{k}'_\alpha| < 2} \left[(-1)^{\bar{\mathbf{p}}'_x} e^{i\varphi} + (-1)^{\bar{\mathbf{p}}_x} e^{-i\varphi} \right] \lambda(\mathbf{p}, \mathbf{p}') \sum_{\sigma} C_{\sigma\mathbf{p}}^+ C_{-\sigma\mathbf{k}}^+ C_{-\sigma\mathbf{p}'} C_{\sigma\mathbf{k}'}.\end{aligned}\quad (22)$$

Obviously, $\hat{H}_t^{(2)}$ also does not contribute to Josephson current in macroscopic systems. The Hamiltonian $\hat{H}_t^{(3)} \sim \psi, \psi^*$ describes transition between coupled and uncoupled states, and will not be reproduced here due to its cumbersomeness. It does not contribute to Josephson current either.

Now the fourth term $\hat{H}_t^{(4)} \sim \psi^* \psi$ describes a transition that is accompanied by production of a coupled electron pair above the upper limit of the gap, and production of a coupled hole pair below the lower limit of the superconductivity gap of the electron system:

$$\begin{aligned}\hat{H}_t^{(4)} &= N_{0x}^{-1} \Omega^{-1} \\ &\quad \times \sum_{\mathbf{p}, \mathbf{k}, \mathbf{p}', \mathbf{k}'} \sum_{\sigma, \mathbf{q}} (-1)^{\bar{\mathbf{k}}_x + \bar{\mathbf{q}}_x} \left[\begin{aligned} &(-1)^{\bar{\mathbf{p}}'_x + \bar{\mathbf{k}}'_x} e^{i\varphi} \\ &+ (-1)^{\bar{\mathbf{p}}_x + \bar{\mathbf{k}}_x} e^{-i\varphi} \end{aligned} \right] \lambda \left(\begin{array}{c} \mathbf{p} - \mathbf{q}, \\ \mathbf{p}' + \mathbf{k}' - \mathbf{k} - \mathbf{q} \end{array} \right) \left[\begin{array}{c} \psi^{\mathbf{k}, \mathbf{p}^*}(\mathbf{q}) \times \\ \psi^{\mathbf{k}', \mathbf{p}'}(\mathbf{k} - \mathbf{k}' + \mathbf{q}) \end{array} \right] C_{\sigma\mathbf{p}}^+ C_{-\sigma\mathbf{k}}^+ C_{-\sigma\mathbf{k}'} C_{\sigma\mathbf{p}'}\end{aligned}\quad (23)$$

while $|\mathbf{p}_\alpha - \mathbf{k}_\alpha| < 2$, $|\mathbf{p}'_\alpha - \mathbf{k}'_\alpha| < 2$.

The Hamiltonian $\hat{H}_t^{(4)}$ can contribute to the Josephson current of a macroscopic system in the second perturbation theory order over λ . In order to demonstrate this, let us consider the transitions defined by this Hamiltonian more in detail, while bearing in mind that the total pair momentum along the directions y and z is an integral of motion. First, consider the case when $\bar{\mathbf{p}}_y = \bar{\mathbf{k}}_y = \bar{\mathbf{p}}'_y = \bar{\mathbf{k}}'_y$ and $\bar{\mathbf{p}}_z = \bar{\mathbf{k}}_z = \bar{\mathbf{p}}'_z = \bar{\mathbf{k}}'_z$, respectively, i.e. the pair momenta are located at the same node in the yz plane of the momentum space. Two coupled states with odd total quantum number (QN) $2\bar{\mathbf{p}}'_x + 1$ can be produced: $C_{\uparrow, \{\bar{\mathbf{p}}'_x, \bar{\mathbf{p}}'_y, \bar{\mathbf{p}}'_z\}}^+ C_{\downarrow, \bar{\mathbf{p}}'_x+1, \bar{\mathbf{p}}'_y, \bar{\mathbf{p}}'_z}^+ |0\rangle$ and $C_{\uparrow, \{\bar{\mathbf{p}}'_x+1, \bar{\mathbf{p}}'_y, \bar{\mathbf{p}}'_z\}}^+ C_{\downarrow, \bar{\mathbf{p}}'_x, \bar{\mathbf{p}}'_y, \bar{\mathbf{p}}'_z}^+ |0\rangle$. In the same way, two coupled states can be produced with odd total QN $2\bar{\mathbf{p}}_x + 1$: $C_{\uparrow, \{\bar{\mathbf{p}}_x, \bar{\mathbf{p}}_y, \bar{\mathbf{p}}_z\}}^+ C_{\downarrow, \{\bar{\mathbf{p}}_x+1, \bar{\mathbf{p}}_y, \bar{\mathbf{p}}_z\}}^+ |0\rangle$ and $C_{\uparrow, \{\bar{\mathbf{p}}_x+1, \bar{\mathbf{p}}_y, \bar{\mathbf{p}}_z\}}^+ C_{\downarrow, \bar{\mathbf{p}}_x, \bar{\mathbf{p}}_y, \bar{\mathbf{p}}_z}^+ |0\rangle$. Therefore, there exist 4 parity-conserving transitions $2\bar{\mathbf{p}}'_x + 1 \rightarrow 2\bar{\mathbf{p}}_x + 1$, and the squared modulus of the corresponding matrix element has a factor of 1/4 related to $\psi^{\mathbf{p}, \mathbf{k}}(\mathbf{q})$ normalization at $\mathbf{p} \neq \mathbf{k}$. In this way, the produced integral factor for energy change in the second order of perturbation theory over λ in $2\bar{\mathbf{p}}'_x + 1 \rightarrow 2\bar{\mathbf{p}}_x + 1$ transitions turns out to be equal to 1.

For each of the total QNs $2\bar{\mathbf{p}}'_x$ and $2\bar{\mathbf{p}}_x$, there exists one coupled state: $C_{\uparrow, \{\bar{\mathbf{p}}'_x, \bar{\mathbf{p}}'_y, \bar{\mathbf{p}}'_z\}}^+ C_{\downarrow, \{\bar{\mathbf{p}}'_x, \bar{\mathbf{p}}'_y, \bar{\mathbf{p}}'_z\}}^+ |0\rangle$ and $C_{\uparrow, \{\bar{\mathbf{p}}_x, \bar{\mathbf{p}}_y, \bar{\mathbf{p}}_z\}}^+ C_{\downarrow, \{\bar{\mathbf{p}}_x, \bar{\mathbf{p}}_y, \bar{\mathbf{p}}_z\}}^+ |0\rangle$, respectively. Therefore, there exists one transition $2\bar{\mathbf{p}}'_x \rightarrow 2\bar{\mathbf{p}}_x$; taking into account the normalization of the function $\psi^{\mathbf{p}, \mathbf{p}}(\mathbf{q})$, it also contributes to the said integral factor, its contribution also being equal to 1.

Parity-violating transitions, symbolically expressed as $2\bar{\mathbf{p}}'_x + 1 \rightarrow 2\bar{\mathbf{p}}_x$ and $2\bar{\mathbf{p}}'_x \rightarrow 2\bar{\mathbf{p}}_x + 1$, are four; their squared matrix elements have factors of 1/2, and the contributions of parity-violating transitions to the integral factor is equal to -2. Thus, contributions of parity-conserving and parity-violating transitions to the integral factor cancel each other when the pair momenta are located at the same node of the yz plane in the momentum space.

Consider now the case where the momenta of the coupled pairs are located at neighboring nodes of the yz plane in the momentum space. These are best considered together as in this case, there is no difference in normalizations of coupled wave functions $\psi^{\mathbf{p}, \mathbf{k}}(\mathbf{q})$, because always, $\mathbf{p} \neq \mathbf{k}$. Assume that there are m initial and final states with odd values of the QN under consideration, and, respectively, n initial and final states with even QN values in the corresponding unit intervals of the momentum space. Then, the number of parity-conserving transitions is $m^2 + n^2$, and that of parity-violating transitions, $2mn$. Therefore, if $m \neq n$ then the number of parity-conserving transitions would be larger than that of parity-violating transitions. On the other hand, evidently, $m = 2n$ (which is confirmed by the above detailed analysis of the particular case where the couple momenta are located at the same node of the yz plane in the momentum space). Thus, the contribution to energy change is larger from parity-conserving transitions than from parity-violating transitions in the case where the momenta of the coupled pairs are located in the neighboring nodes of the yz plane in the momentum space. Consequently, an uncompensated Josephson current is produced, and the ground state of the system is a state with $\varphi = 0$, i.e. it is a normal Josephson junction. Besides, the presence of the term $\psi^{\mathbf{k}, \mathbf{p}^*}(\mathbf{q}) \times \psi^{\mathbf{k}', \mathbf{p}'}(\mathbf{k} - \mathbf{k}' + \mathbf{q})$ in the right-hand part of (23) allows one to conclude that the Josephson current density tends to zero as the pair coupling energy E_b , linearly related to the energy gap as $\Delta = 7E_b$ [13], decreases, which agrees with the experimental data too.

IV. JOSEPHSON EFFECT IN THE BCS THEORY

Consider a superconductor at zero temperature from the point of view of the BCS theory. In this system, the ground state $|BCS\rangle$ satisfies the following relation:

$$b_{\sigma\mathbf{k}}|BCS\rangle = 0, \quad (24)$$

where $b_{\sigma\mathbf{k}}$ are Bogoliubov quasi-particle annihilation operators related to electron creation and annihilation operators in the following way (taking $\sigma = \pm 1/2$):

$$\begin{aligned} b_{\sigma\mathbf{k}}^+ &= u_{0\mathbf{k}}c_{\sigma\mathbf{k}}^+ - 2\sigma v_{0\mathbf{k}}c_{-\sigma\mathbf{k}}, \\ b_{\sigma\mathbf{k}} &= u_{0\mathbf{k}}c_{\sigma\mathbf{k}} - 2\sigma v_{0\mathbf{k}}c_{-\sigma\mathbf{k}}^+. \end{aligned} \quad (25)$$

We assume here that Bogoliubov transformation parameters $u_{0\mathbf{k}}$ and $v_{0\mathbf{k}}$ are real values, i.e. in terms of the BCS theory, the superconductor wave function phase is zero. Take now a unitary transformation of the basis set of wave functions produced using the $|BCS\rangle$ state and Bogoliubov quasi-particle operators with the operator $\exp(i\varphi\hat{N})$, where \hat{N} is the operator of total number of electrons in the system ($\hat{N} = \sum_{\sigma, \mathbf{k}} c_{\sigma\mathbf{k}}^+ c_{\sigma\mathbf{k}}$). Now, the ground state of the system is $|BCS'\rangle = \exp(i\varphi\hat{N})|BCS\rangle$. It can easily be seen after a transition to new Bogoliubov quasi-particle

operators $b_{\sigma\mathbf{k}}^+$ and $b'_{\sigma\mathbf{k}}$:

$$\begin{aligned} b_{\sigma\mathbf{k}}^+ &= u_{\mathbf{k}} c_{\sigma\mathbf{k}}^+ - 2\sigma v_{\mathbf{k}} c_{-\sigma\mathbf{k}}, \\ b'_{\sigma\mathbf{k}} &= u_{\mathbf{k}}^* c_{\sigma\mathbf{k}} - 2\sigma v_{\mathbf{k}}^* c_{-\sigma\mathbf{k}}, \end{aligned} \quad (26)$$

where

$$\begin{aligned} u_{\mathbf{k}} &= u_{0\mathbf{k}} e^{i\varphi}, \\ v_{\mathbf{k}} &= v_{0\mathbf{k}} e^{-i\varphi}. \end{aligned} \quad (27)$$

Obviously, the new ground state $|BCS'\rangle$ of the superconductor satisfies the relation:

$$b'_{\sigma\mathbf{k}} |BCS'\rangle = 0, \quad (28)$$

since $b'_{\sigma\mathbf{k}} = \exp(i\varphi\hat{N}) b_{\sigma\mathbf{k}} \exp(-i\varphi\hat{N})$. The superconductor wave function in the BCS representation acquires then a phase 2φ , since $u_{\mathbf{k}} v_{\mathbf{k}}^* \sim \exp(2i\varphi)$. *It is important to note that the transformation $\exp(i\varphi\hat{N})$ does not correspond to any physical effect on the system.*

From the point of view of quantum physics, no transformation of the basis set of wave functions can result in changes in observable values, e.g. the system energy spectrum, as the matrix elements corresponding to these values change as well. This, however, is not quite correct in the BCS theory.

Indeed, consider two superconductors linked to each other by the Hamiltonian H_T :

$$H_T = \sum_{\sigma, \mathbf{k}, \mathbf{q}} T_{\mathbf{k}\mathbf{q}} c_{\sigma\mathbf{k}}^+ c_{\sigma\mathbf{q}} + H.c. \quad (29)$$

We assume the operators corresponding to momentum \mathbf{k} to be related to the electrons of the first superconductor, and those corresponding to momentum \mathbf{q} to be related to the electrons of the second superconductor, respectively. Applying the transformation $\exp(i\varphi_1\hat{N}_1)$ to the basis wave function set of the first superconductor, and the transformation $\exp(i\varphi_2\hat{N}_2)$ respectively, to the basis set of the second one, and expressing the operators c^+ and c in terms of Bogoliubov operators

$$\begin{aligned} c_{\sigma\mathbf{p}}^+ &= u_{\mathbf{p}}^* b'_{\sigma\mathbf{p}}^+ + 2\sigma v_{\mathbf{p}} b'_{-\sigma\mathbf{p}}, \\ c_{\sigma\mathbf{p}} &= u_{\mathbf{p}} b'_{\sigma\mathbf{p}} + 2\sigma v_{\mathbf{p}}^* b'_{-\sigma\mathbf{p}}, \end{aligned} \quad (30)$$

where $\mathbf{p} = \mathbf{k}, \mathbf{q}$, allows calculating the second-order correction δE_2 to the energy of the two-superconductor system:

$$\delta E_2 = -2 \sum_{\mathbf{k}, \mathbf{q}} |T_{\mathbf{k}\mathbf{q}}|^2 \frac{|v_{\mathbf{k}} u_{\mathbf{q}} + u_{\mathbf{k}} v_{\mathbf{q}}|^2}{E_{\mathbf{k}} + E_{\mathbf{q}}}, \quad (31)$$

which is absolutely identical to the expression provided by Anderson for binding energy of two superconductors at zero temperature (see Eq.(3) in [2]). This expression is the initial equation in Josephson current calculations according to the BCS theory.

The right-hand part of (31) depends on phase difference $\varphi_2 - \varphi_1$ between the two superconductors, which contradicts the principles of quantum mechanics, since the observed values should not depend on the chosen basis wave function set of the system. It should be noted once again that the transformation $\exp(i\varphi_j\hat{N}_j)$ ($j = 1, 2$) of the basis set of two superconductors does not correspond to any physical effect. Therefore, the expression for Josephson current used in the BCS theory contradicts the principles of quantum mechanics.

At this point, one might conclude examining the BCS theory interpretation of the Josephson effect, leaving the experts on the BSC theory to their own devices in their search for a way to resolve the appearing paradox. Let us, however, try and assist our colleagues in solving this tangled problem. In order to achieve this goal we must return to the fundamentals of the BCS theory.

In the BCS theory the Hamiltonian of the system can be reduced to the form $\hat{H}_0 = \sum_{\sigma, \mathbf{p}} E_{\mathbf{p}} b'_{\sigma\mathbf{p}}^+ b'_{\sigma\mathbf{p}}$ with the use of a variation procedure [21]. This variation procedure is interpreted by the BCS specialists as a generalization of the Hartree-Fock approach for description of the single particle spectrum of the system and enables the use of linear combinations of wave functions with various numbers of electrons for the ground state of the system. Of course, we could agree with such an interpretation, would there be no such an important objection as the following: the

Hamiltonian \hat{H}'_0 does not preserve the total number of the electrons in the system, i.e. one of the main system's symmetries is broken. In such a situation the appearance of the above-mentioned paradox in case of the use of the Hamiltonian \hat{H}'_0 as the principle Hamiltonian in the perturbation theory is not surprising. It is noteworthy to notice here that when building various models for solids including those based on the Hartree-Fock approximation physicists strictly observe the requirements imposed on the symmetry of the system, that is why the interpretation of the Hartree-Fock approximation in the BCS theory cannot be called felicitous.

Of course, the interpretation of the Hartree-Fock approximation suggested by the authors of the BCS theory is not the only possible one. Obviously, the principle of system description we had earlier put forward [15] also is a generalization of the Hartree-Fock approximation, which, however, can be used to describe *multi*-particle spectra of the system. The advantage of our version of the generalization is not reduced to the absence of the above-mentioned paradox, this version also provides the possibility of an arbitrarily precise description with extension of the right-hand part of the (12); a precise description of a system containing N_0 electrons would only require N_0 terms to be used in the right-hand part of (12). One should recall in this connection that all consistent physical theories, with the apparent exception of the BCS theory, are required to provide an arbitrarily precise description of the corresponding systems. To our knowledge, at least, there are no studies where this problem were consistently studied in the framework of this theory.

The above considerations suggest a way to resolving the paradox related to the phase-difference dependence present in the right-hand part of Eq. (31). If our opponents do not for some reasons agree with the suggested explanation of the paradox, we would, of course, let them find their own way to eliminate this dependence in order to keep the principles of quantum mechanics inviolable. As for the conclusions of this study, the very fact of existence of the paradox is what is of importance.

V. CONCLUSIONS.

In my opinion, the process of acknowledgement of Josephson's and Anderson's work results by the academic community is not quite consistent. Indeed, if we are to acknowledge their interpretation of the effect under discussion, then the interpretation of the basic concepts of quantum mechanics should be changed, to supplement it with a fundamentally new postulate of the dependence of measurement results on the selected representation of the wave function of the quantum object. In this case, it should also be admitted that modern renderings of quantum physics undeservedly omit the references to this contribution of the eminent scholars, which makes them, strictly speaking, incorrect.

Assuming on the other hand that the Josephson effect might have a different interpretation that does not require changing the bases of quantum physics, such as for example put forward in the present paper, one should reject a vast amount of interpretations of experimental data produced in the framework of the BCS theory. It should be reminded in this context that the problem of the simplest BCS Hamiltonian cannot be solved using the quasi-spin operator methods either [22], which is why all the results of the theory have been obtained using the anomalous expectation values technique or the equivalent Bogoliubov operator technique.

It appears that the academic community should either complete the process of acknowledgement of the BCS theory or request the physicists to create a different theory of superconductivity.

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